VII. *On the Means of diſcovering the Diſtance, Magnitude, &c. of the Fixed Stars, in conſequence of the Diminution of the Velocity of their Light, in caſe ſuch a Diminution ſhould be found to take place in any of them, and ſuch other Data ſhould be procured from Obſervations, as would be farther neceſſary for that Purpoſe. By the Rev.* John Michell, *B. D. F. R. S. In a Letter to* Henry Cavendiſh, *Eſq. F. R. S. and A. S.*

Read November 27, 1783.

DEAR SIR, Thornhill, May 26, 1783.

THE method, which I mentioned to you when I was laſt in London, by which it might perhaps be poſſible to find the diſtance, magnitude, and weight of ſome of the fixed ſtars, by means of the diminution of the velocity of their light, occurred to me ſoon after I wrote what is mentioned by Dr. PRIESTLEY in his Hiſtory of Optics, concerning the diminution of the velocity of light in conſequence of the attraction of the ſun; but the extreme difficulty, and perhaps impoſſibility, of procuring the other data neceſſary for this purpoſe appeared to me to be ſuch objections againſt the ſcheme, when I firſt thought of it, that I gave it then no farther conſideration. As ſome late obſervations, however, begin to give us a little more chance of procuring ſome at leaſt of theſe data, I thought it would not be amiſs, that aſtronomers ſhould be apprized of the method, I propoſe (which, as far as I know,

has

has not been ſuggeſted by any one elſe) leſt, for want of being aware of the uſe, which may be made of them, they ſhould neglect to make the proper obſervations, when in their power; I ſhall therefore beg the favour of you to preſent the following paper on this ſubject to the Royal Society.

I am, &c.

THE very great number of ſtars that have been diſcovered to be double, triple, &c. particularly by Mr. HERSCHEL *, if we apply the doctrine of chances, as I have heretofore done in my "Enquiry into the probable Parallax, &c. of the Fixed "Stars," publiſhed in the Philoſophical Tranſactions for the year 1767, cannot leave a doubt with any one, who is properly aware of the force of thoſe arguments, that by far the greateſt part, if not all of them, are ſyſtems of ſtars ſo near to each other, as probably to be liable to be affected ſenſibly by their mutual gravitation; and it is therefore not unlikely, that the periods of the revolutions of ſome of theſe about their principals (the ſmaller ones being, upon this hypotheſis, to be conſidered as ſatellites to the others) may ſome time or other be diſcovered.

2. Now the apparent diameter of any central body, round which any other body revolves, together with their apparent diſtance from each other, and the periodical time of the revolving

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* See his Catalogue of Stars of this kind, publiſhed in the Philoſophical Tranſactions for the year 1782, which is indeed a moſt valuable preſent to the aſtronomical world. By a happy application of very high magnifyiug powers to his teleſcopes, and by a moſt perſevering induſtry in obſerving, he has made a very wonderful progreſs in this branch of aſtronomy, in which almoſt nothing of any conſequence had been done by any one before him.

ing body being given, the denſity of the central body will be given likewiſe. See Sir ISAAC NEWTON's Prin. b. III. pr. VIII. cor. 1.

3. But the denſity of any central body being given, and the velocity any other body would acquire by falling towards it from an infinite height, or, which is the ſame thing, the velocity of a comet revolving in a parabolic orbit, at its ſurface, being given, the quantity of matter, and conſequently the real magnitude of the central body, would be given likewiſe.

4. Let us now ſuppoſe the particles of light to be attracted in the ſame manner as all other bodies with which we are acquainted; that is, by forces bearing the ſame proportion to their *vis inertiæ*, of which there can be no reaſonable doubt, gravitation being, as far as we know, or have any reaſon to believe, an univerſal law of nature. Upon this ſuppoſition then, if any one of the fixed ſtars, whoſe denſity was known by the above-mentioned means, ſhould be large enough ſenſibly to affect the velocity of the light iſſuing from it, we ſhould have the means of knowing its real magnitude, &c.

5. It has been demonſtrated by Sir ISAAC NEWTON, in the 39th propoſition of the firſt book of his Principia, that if a right line be drawn, in the direction of which a body is urged by any forces whatſoever, and there be erected at right angles to that line perpendiculars every where proportional to the forces at the points, at which they are erected reſpectively, the velocity acquired by a body beginning to move from reſt, in conſequence of being ſo urged, will always be proportional to the ſquare root of the area deſcribed by the aforeſaid perpendiculars. And hence,

6. If ſuch a body, inſtead of beginning to move from reſt, had already ſome velocity in the direction of the ſame line, when

when it began to be urged by the aforeſaid forces, its velocity would then be always proportional to the ſquare root of the ſum or difference of the aforeſaid area, and another area, whoſe ſquare root would be proportional to the velocity which the body had before it began to be ſo urged; that is, to the ſquare root of the ſum of thoſe areas, if the motion acquired was in the ſame direction as the former motion, and the ſquare root of the difference, if it was in a contrary direction. See cor. 2. to the aboveſaid propoſition.

7. In order to find, by the foregoing propoſition, the velocity which a body would acquire by falling towards any other central body, according to the common law of gravity, let C in the figure (tab. III.) repreſent the centre of the central body, towards which the falling body is urged, and let CA be a line drawn from the point C, extending infinitely towards A. If then the line RD be ſuppoſed to repreſent the force, by which the falling body would be urged at any point D, the velocity which it would have acquired by falling from an infinite height to the place D would be the ſame as that which it would acquire by falling from D to C with the force RD, the area of the infinitely extended hyperbolic ſpace ADRB, where RD is always inverſely proportional to the ſquare of DC, being equal to the rectangle RC contained between the lines RD and CD. From hence we may draw the following corollaries.

8. Cor. 1. The central body DEF remaining the ſame, and conſequently the forces at the ſame diſtances remaining the ſame likewiſe, the areas of the rectangles RC, *r*C will always be inverſely as the diſtances of the points D, *d* from C, their ſides RD, *rd* being inverſely in the duplicate ratio of the ſides CD, C*d*: and therefore, becauſe the velocity of a body falling from an infinite height towards the point C, is always in the

ſub-duplicate ratio of theſe rectangles, it will be in the ſub-duplicate ratio of the lines CD, C*d* inverſely. Accordingly the velocities of comets revolving in parabolic orbits are always in the ſub-duplicate ratio of their diſtances from the ſun inverſely; and the velocities of the planets, at their mean diſtances (being always in a given ratio to the velocity of ſuch comets, *viz.* in the ſub-duplicate ratio of 1 to 2) muſt neceſſarily obſerve the ſame law likewiſe.

9. Cor. 2. The magnitude of the central body remaining the ſame, the velocity of a body falling towards it from an infinite height will always be, at the ſame diſtance from the point C, taken any where without the central body, in the ſub-duplicate ratio of its denſity; for in this caſe the diſtance C*d* will remain the ſame, the line *rd* only being increaſed or diminiſhed in the proportion of the denſity, and the rectangle *r*C conſequently increaſed or diminiſhed in the ſame proportion.

10. Cor. 3. The denſity of the central body remaining the ſame, the velocity of a body falling towards it from an infinite height will always be as its ſemi-diameter, when it arrives at the ſame proportional diſtance from the point C; for the weights, at the ſurfaces of different ſphæres of the ſame denſity are as their reſpective ſemi-diameters; and therefore the ſides RD and CD, or any other ſides *rd* and C*d*, which are in a given ratio to thoſe ſemi-diameters, being both increaſed or diminiſhed in the ſame proportion, the rectangles RC or *r*C will be increaſed or diminiſhed in the duplicate ratio of the ſemi-diameter CD, and conſequently the velocity in the ſimple ratio of CD.

11. Cor. 4. If the velocity of a body falling from an infinite height towards different central bodies is the ſame, when it arrives at their ſurfaces, the denſity of thoſe central bodies muſt be

in the duplicate ratio of their ſemi-diameters inverſely; for by the laſt cor. the denſity of the central body remaining the ſame, the rectangle RC will be in the duplicate ratio of CD; in order therefore that the rectangle RC may always remain the ſame, the line RD muſt be inverſely, as CD, and conſequently the denſity inverſely, as the ſquare of CD.

12. Cor. 5. Hence the quantity of matter contained in thoſe bodies muſt be in the ſimple ratio of their ſemi-diameters directly; for the quantity of matter being always in a ratio compounded of the ſimple ratio of the denſity, and the triplicate ratio of their ſemi-diameters, if the denſity is in the inverſe duplicate ratio of the ſemi-diameters, this will become the direct triplicate and inverſe duplicate, that is, when the two are compounded together, the ſimple ratio of the ſemi-diameters,

13. The velocity a body would acquire by falling from an infinite height towards the ſun, when it arrived at his ſurface, being, as has been ſaid before in article 3d, the ſame with that of a comet revolving in a parabolic orbit in the ſame place, would be about 20,72 times greater than that of the earth in its orbit at its mean diſtance from the ſun; for the mean diſtance of the earth from the ſun, being about 214,64 of the ſun's ſemidiameters, the velocity of ſuch a comet would be greater at that diſtance than at the diſtance of the earth from the ſun, in the ſub-duplicate ratio of 214,64 to 1, and the velocity of the comet being likewiſe greater than that of planets, at their mean diſtances, in the ſub-duplicate ratio of 2 to 1; theſe, when taken together, will make the ſub-duplicate ratio of 429,28 to 1, and the ſquare root of 429,28 is 20,72, very nearly.

14. The ſame reſult would have been obtained by taking the line RD proportional to the force of gravity at the ſun's ſurface, and DC equal to his ſemi-diameter, and from thence computing a velocity, which ſhould be proportional to the ſquare root of the area RC when compared with the ſquare root of another area, one of whoſe ſides ſhould be proportional to the force of gravity at the ſurface of the earth; and the other ſhould be, for inſtance, equal to 16 feet, 1 inch, the ſpace a body would fall through in one ſecond of time, in which caſe it would acquire a velocity of 32 feet, 2 inches per ſecond. The velocity thus found compared with the velocity of the earth in its orbit, when computed from the ſame elements, neceſſarily gives the ſame reſult. I have made uſe of this latter method of computation upon a former occaſion, as may be ſeen in Dr. PRIESTLEY's Hiſtory of Optics, p. 787, &c. but I have rather choſen to take the velocity from that of a comet, in the article above, on account of its greater ſimplicity, and its more immediate connexion with the ſubject of this paper.

15. The velocity of light, exceeding that of the earth in its orbit, when at its mean diſtance from the ſun, in the proportion of about 10.310 to 1, if we divide 10.310 by 20,72, the quotient 497, in round numbers, will expreſs the number of times, which the velocity of light exceeds the velocity a body could acquire by falling from an infinite height towards the ſun, when it arrived at his ſurface; and an area whoſe ſquare root ſhould exceed the ſquare root of the area RC, where RD is ſuppoſed to repreſent the force of gravity at the ſurface of the ſun, and CD is equal to his ſemi-diameter, in the ſame proportion, muſt conſequently exceed the area RC in the proportion of 247.009, the ſquare of 497 to 1.

16. Hence, according to article 10, if the ſemi-diameter of a ſphære of the ſame denſity with the ſun were to exceed that of the ſun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its ſurface a greater velocity than that of light, and conſequently, ſuppoſing light to be attracted by the ſame force in proportion to its vis inertiæ, with other bodies, all light emitted from ſuch a body would be made to return towards it, by its own proper gravity.

17. But if the ſemi-diameter of a ſphære, of the ſame denſity with the ſun, was of any other ſize leſs than 497 times that of the ſun, though the velocity of the light emitted from ſuch a body, would never be wholly deſtroyed, yet would it always ſuffer ſome diminution, more or leſs, according to the magnitude of the ſaid ſphære; and the quantity of this diminution may be eaſily found in the following manner: Suppoſe S to repreſent the ſemi-diameter of the ſun, and aS to repreſent the ſemi-diameter of the propoſed ſphære; then, as appears from what has been ſhewn before, the ſquare root of the difference between the ſquare of 497 S and the ſquare of aS will be always proportional to the ultimately remaining velocity, after it has ſuffered all the diminution, it can poſſibly ſuffer from this cauſe; and conſequently the difference between the whole velocity of light, and the remaining velocity, as found above, will be the diminution of its velocity. And hence the diminution of the velocity of light emitted from the ſun, on account of it's gravitation towards that body, will be ſomewhat leſs than a 494.000dth part of the velocity which it would have had if no ſuch diminution had taken place; for the ſquare of 497 being 247.009, and the ſquare of 1 being 1, the diminution of the velocity will be the difference between

the ſquare root of 247.009, and the ſquare root of 247.008, which amounts, as above, to ſomewhat leſs than one 494.000th part of the whole quantity.

18. The ſame effects would likewiſe take place, according to article 11, if the ſemi-diameters were different from thoſe mentioned in the two laſt articles, provided the denſity was greater or leſs in the duplicate ratio of thoſe ſemi-diameters inverſely.

19. The better to illuſtrate this matter, it may not be amiſs to take a particular example. Let us ſuppoſe then, that it ſhould appear from obſervations made upon ſome one of thoſe double ſtars above alluded to, that one of the two performed its revolution round the other in 64 years, and that the central one was of the ſame denſity with the ſun, which it muſt be, if its apparent diameter, when ſeen from the other body, was the ſame as the apparent diameter of the ſun would be if ſeen from a planet revolving round him in the ſame period: let us further ſuppoſe, that the velocity of the light of the central body was found to be leſs than that of the ſun, or other ſtars whoſe magnitude was not ſufficient to affect it ſenſibly, in the proportion of 19 to 20. In this caſe then, according to article 17, the ſquare root of 247.009 SS muſt be to the ſquare root of the difference between 247.009 SS and aaSS as 20 to 19. But the ſquares of 20 and 19 being 400 and 361, the quantity 247.009 SS muſt therefore be to the difference between this quantity and aaSS in the ſame proportion, that is as 247.009 to 222.925,62; and aaSS muſt conſequently be equal to 24.083, 38 SS, whoſe ſquare root 155,2 S nearly, or, in round numbers, 155 times the diameter of the ſun, will be the diameter of the central ſtar ſought.

20. As the ſquares of the periodical times of bodies, revolving round a central body, are always proportional to the cubes of their mean diſtances, the diſtance of the two bodies from each other muſt therefore, upon the foregoing ſuppoſitions, be ſixteen times greater in proportion to the diameter of the central body, than the diſtance of the earth from the ſun in proportion to his diameter; and that diameter being already found to be alſo greater than that of the ſun in the proportion of 155,2 to 1, this diſtance will conſequently be greater than that of the earth and ſun from each other in the proportion of 16 times 155,2, that is 2483,2 to 1.

21. Let us farther ſuppoſe, that from the obſervations, the greateſt diſtance of the two ſtars in queſtion appeared to be only one ſecond; we muſt then multiply the number 2483,2 by 206.264,8, the number of ſeconds in the radius of a circle, and the product 512.196.750 will ſhew the number of times which ſuch a ſtar's diſtance from us muſt exceed that of the ſun. The quantity of matter contained in ſuch a ſtar would be $\overline{155{,}2}^{3}$ or 3.738.308 times as much as that contained in the ſun; its light, ſuppoſing the ſun's light to take up 8′. 7″. in coming to the earth, would, with its common velocity, require 7.900 years to arrive at us, and 395 years more on account of the diminution of that velocity; and ſuppoſing ſuch a ſtar to be equally luminous with the ſun, it would ſtill be very ſufficiently viſible, I apprehend, to the naked eye, notwithſtanding its immenſe diſtance.

22. In the elements which I have employed in the above computations, I have ſuppoſed the diameter of the central ſtar to have been obſerved, in order to aſcertain its denſity, which cannot be known without it; but the diameter of ſuch a ſtar is

much too ſmall to be obſerved by any teleſcopes yet exiſting, or any that it is probably in the power of human abilities to make; for the apparent diameter of the central ſtar, if of the ſame denſity with the ſun, when ſeen from another body, which would revolve round it in 64 years, would be only the 1717th part of the diſtance of thoſe bodies from each other, as will appear from multiplying 107,32, the number of times the ſun's diameter is contained in his diſtance from the earth, by 16, the greater proportional diſtance of the revolving body, correſponding to 64 years inſtead of 1. Now the 1717th part of a ſecond muſt be magnified 309.060 times in order to give it an apparent diameter of three minutes; and three minutes, if the teleſcopes were mathematically perfect, and there was no want of diſtinctneſs in the air, would be but a very ſmall matter to judge of*.

23. But

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* In Mr. HERSCHEL's Obſervations upon the Fixed Stars abovementioned, almoſt all of them are repreſented as appearing with a well-defined round diſc. That this is not the real diſc, but only an optical appearance, occaſioned perhaps by the conſtitution of the eye, when the pencil, by which objects are ſeen, is ſo exceedingly ſmall as thoſe which he employed upon this occaſion, is very manifeſt, from the obſervations themſelves, of which indeed Mr. HERSCHEL ſeems to be himſelf ſufficiently aware: if it were not ſo, the intenſity of the light of theſe ſtars muſt either be exceedingly inferior indeed to that of the ſun, or they muſt be immenſely larger, otherwiſe they muſt have a very ſenſible parallax; for the ſun, if removed to 10.000.000 times his preſent diſtance, would ſtill, I apprehend, be of about the brightneſs of the ſtars of the ſixth magnitude; in which caſe he muſt be magnified 1.000.000 times to make his apparent diſc of any ſenſible magnitude; or, on the other hand, if he was only removed to a thouſandth part of that diſtance, then he muſt be leſs luminous in the proportion of 1.000.000 to 1, to make him appear no brighter than a ſtar of the ſixth magnitude. Now the ſun's diameter being contained nearly 215 times in the diameter of the earth's orbit, the annual parallax therefore of ſuch a body in that caſe, if it was placed in the pole of the ecliptic,

23. But though there is not the leaſt probability that this element, ſo eſſential to be known, in order to determine with preciſion the exact diſtance and magnitude of a ſtar, can ever be obtained, where it is in the ſame circumſtances, or nearly the ſame, with thoſe above ſuppoſed, yet the other elements, ſuch as perhaps may be obtained, are ſufficient to determine the diſtance, &c. with a good deal of probability, within ſome moderate limits; for in whatever ratio the real diſtance of the two ſtars may be greater or leſs than the diſtance ſuppoſed, the denſity of the central ſtar muſt be greater or leſs in the ſixth power of that ratio inverſely; for the periodic time of the revolving body being given, the quantity of matter contained in the central body muſt be as the cube of their diſtance from each other. See Sir I. Newton's Prin. b. 3d. pr. 8th. cor 3d. But the quantity of matter in different bodies, at whoſe ſurfaces the velocity acquired by falling from an infinite height is the ſame, muſt be, according to art. 12, directly as their ſemi-diameters; the ſemi-diameters therefore of ſuch bodies muſt be in the triplicate ratio of the diſtance of the revolving body; and conſequently their denſities, by art. 11, being in the inverſe duplicate ratio of their ſemi-diameters, muſt be in the inverſe ſextuplicate ratio of the diſtance of the revolving body. Hence if the real diſtance ſhould be greater or leſs than that ſuppoſed, in the proportion of two or three to one, the denſity of the central body muſt be leſs or greater, in the firſt caſe, in the proportion of 64, or in the latter of 729 to 1.

ecliptic, would be 215 times its apparent diameter; and as the bright ſtar in Lyrâ appeared to Mr. Herschel about a third part of a ſecond in diameter, if this was its real diſc, and it was no bigger than the ſun, it would conſequently have an annual parallax in the pole of the ecliptic of about 72″.

24. There is alſo another circumſtance, from which perhaps ſome little additional probability might be derived, with regard to the real diſtance of a ſtar, ſuch as that we have ſuppoſed; but upon which however, it muſt be acknowledged, that no great ſtreſs can be laid, unleſs we had ſome better analogy to go upon than we have at preſent. The circumſtance I mean is the greater ſpecific brightneſs which ſuch a ſtar muſt have, in proportion as the real diſtance is leſs than that ſuppoſed, and *vice verſâ*; ſince, in order that the ſtar may appear equally luminous, its ſpecific brightneſs muſt be as the fourth power of its diſtance inverſely; for the diameter of the central ſtar being as the cube of the diſtance between that and the revolving ſtar, and their diſtance from the earth being in the ſimple ratio of their diſtance from each other, the apparent diameter of the central ſtar muſt be as the ſquare of its real diſtance from the earth, and conſequently, the ſurface of a ſphære being as the ſquare of its diameter, the area of the apparent diſc of ſuch a ſtar muſt be as the fourth power of its diſtance from the earth; but in whatever ratio the apparent diſc of the ſtar is greater or leſs, in the ſame ratio inverſely muſt be the intenſity of its light, in order to make it appear equally luminous. Hence, if its real diſtance ſhould be greater or leſs than that ſuppoſed in the proportion of 2 or 3 to 1, the intenſity of its light muſt be leſs or greater, in the firſt caſe, in the proportion of 16, or, in the latter of 81 to 1.

25. According to Monſ. BOUGUER (ſee his Traité d'Optique) the brightneſs of the ſun exceeds that of a wax candle in no leſs a proportion than that of 8000 to 1. If therefore the brightneſs of any of the fixed ſtars ſhould not exceed that of our common candles, which, as being ſomething leſs luminous than

wax, we will ſuppoſe in round numbers to be only one 10.000dth part as bright as the ſun, ſuch a ſtar would not be viſible at more than an 100dth part of the diſtance, at which it would be viſible, if it was as bright as the ſun. Now becauſe the ſun would ſtill appear, I apprehend, as luminous, as the ſtar Sirius, when removed to 400.000 times his preſent diſtance, ſuch a body, if no brighter than our common candles, would only appear equally luminous with that ſtar at 4000 times the diſtance of the ſun, and we might then begin to be able, with the beſt teleſcopes, to diſtinguiſh ſome ſenſible apparent diameter of it; but the apparent diameters of the ſtars of the leſs magnitudes would ſtill be too ſmall to be diſtinguiſhable even with our beſt teleſcopes, unleſs they were yet a good deal leſs luminous, which may poſſibly however be the caſe with ſome of them; for, though we have indeed very ſlight grounds to go upon with regard to the ſpecific brightneſs of the fixed ſtars compared with that of the ſun at preſent, and can therefore only form very uncertain and random conjectures concerning it, yet from the infinite variety which we find in the works of the creation, it is not unreaſonable to ſuſpect, that very poſſibly ſome of the fixed ſtars may have ſo little natural brightneſs in proportion to their magnitude, as to admit of their diameters having ſome ſenſible apparent ſize, when they ſhall come to be more carefully examined, and with larger and better teleſcopes than have been hitherto in common uſe.

26. With regard to the ſun, we know that his whole ſurface is extremely luminous, a very ſmall and temporary interruption ſometimes from a few ſpots only excepted. This univerſal and exceſſive brightneſs of the whole ſurface is probably owing to an atmoſphære, which being luminous throughout,

and in ſome meaſure alſo tranſparent, the light, proceeding from a conſiderable depth of it, all arrives at the eye; in the ſame manner as the light of a great number of candles would do, if they were placed one behind another, and their flames were ſufficiently tranſparent to permit the light of the more diſtant ones to paſs through thoſe that were nearer, without any interruption.

27. How far the ſame conſtitution may take place in the fixed ſtars we don't know; probably however it may do ſo in many; but there are ſome appearances with regard to a few of them, which ſeem to make it probable, that it does not do ſo univerſally. Now, if I am right in ſuppoſing the light of the ſun to proceed from a luminous atmoſphære, which muſt neceſſarily diffuſe itſelf equally over the whole ſurface, and I think there can be very little doubt that this is really the caſe, this conſtitution cannot well take place in thoſe ſtars, which are in ſome degree periodically more and leſs luminous, ſuch as that in Collo Ceti, &c. It is alſo not very improbable, that there is ſome difference from that of the ſun, in the conſtitution of thoſe ſtars, which have ſometimes appeared and ſometimes diſappeared, of which that in the conſtellation of Caſſiopeia is a notable inſtance. And if thoſe conjectures are well founded which have been formed by ſome philoſophers concerning ſtars of theſe kinds, that they are not wholly luminous, or at leaſt not conſtantly ſo, but that all, or by far the greateſt part of their ſurfaces is ſubject to conſiderable changes, ſometimes becoming luminous, and at other times being extinguiſhed; it is amongſt the ſtars of this ſort, that we are moſt likely to meet with inſtances of a ſenſible apparent diameter, their light being much more likely not to be ſo great in proportion as that of the ſun, which, if removed to four hundred thouſand times

his

his preſent diſtance would ſtill appear, I apprehend, as bright as Sirius, as I have obſerved above; whereas it is hardly to be expected, with any teleſcopes whatſoever, that we ſhould ever be able to diſtinguiſh a well defined diſc of any body of the ſame ſize with the ſun at much more than ten thouſand times his diſtance.

28. Hence the greateſt diſtance at which it would be poſſible to diſtinguiſh any ſenſible apparent diameter of a body as denſe as the ſun cannot well greatly exceed five hundred times ten thouſand, that is, five million times the diſtance of the ſun; for if the diameter of ſuch a body was not leſs than five hundred times that of the ſun, its light, as has been ſhewn above, in art. 16. could never arrive at us.

29. If there ſhould really exiſt in nature any bodies, whoſe denſity is not leſs than that of the ſun, and whoſe diameters are more than 500 times the diameter of the ſun, ſince their light could not arrive at us; or if there ſhould exiſt any other bodies of a ſomewhat ſmaller ſize, which are not naturally luminous; of the exiſtence of bodies under either of theſe circumſtances, we could have no information from ſight; yet, if any other luminous bodies ſhould happen to revolve about them we might ſtill perhaps from the motions of theſe revolving bodies infer the exiſtence of the central ones with ſome degree of probability, as this might afford a clue to ſome of the apparent irregularities of the revolving bodies, which would not be eaſily explicable on any other hypotheſis; but as the conſequences of ſuch a ſuppoſition are very obvious, and the conſideration of them ſomewhat beſide my preſent purpoſe, I ſhall not proſecute them any farther.

30. The diminution of the velocity of light, in caſe it ſhould be found to take place in any of the fixed ſtars, is the principal phænomenon whence it is propoſed to diſcover their diſtance, &c. Now the means by which we may find what this diminution amounts to, ſeems to be ſupplied by the difference which would be occaſioned in conſequence of it, in the refrangibility of the light, whoſe velocity ſhould be ſo dimiminiſhed. For let us ſuppoſe with Sir ISAAC NEWTON (ſee his Optics, prop. VI. paragr. 4 and 5) that the refraction of light is occaſioned by a certain force impelling it towards the refracting medium, an hypotheſis which perfectly accounts for all the appearances. Upon this hypotheſis the velocity of light in any medium, in whatever direction it falls upon it, will always bear a given ratio to the velocity it had before it fell upon it, and the ſines of incidence and refraction will, in conſequence of this, bear the ſame ratio to each other with theſe velocities inverſely. Thus, according to this hypotheſis, if the ſines of the angles of incidence and refraction, when light paſſes out of air into glaſs, are in the ratio of 31 to 20, the velocity of light in the glaſs muſt be to its velocity in air in the ſame proportion of 31 to 20. But becauſe the areas, repreſenting the forces generating theſe velocities, are as the ſquares of the velocities, ſee art. 5. and 6. theſe areas muſt be to each other as 961 to 400. And if 400 repreſents the area which correſponds to the force producing the original velocity of light, 561, the difference between 961 and 400, muſt repreſent the area correſponding to the additional force, by which the light was accelerated at the ſurface of the glaſs.

31. In art. 19. we ſuppoſed, by way of example, the velocity of the light of ſome particular ſtar to be diminiſhed in the

ratio of 19 to 20, and it was there obſerved, that the area repreſenting the remaining force which would be neceſſary to generate the velocity 19, was therefore properly repreſented by $\frac{361}{400}$dth parts of the area, that ſhould repreſent the force that would be neceſſary to generate the whole velocity of light, when undiminiſhed. If then we add 561, the area repreſenting the force by which the light is accelerated at the ſurface of the glaſs, to 361, the area repreſenting the force which would have generated the diminiſhed velocity of the ſtar's light, the ſquare root of 922, their ſum, will repreſent the velocity of the light with the diminiſhed velocity, after it has entered the glaſs. And the ſquare root of 922 being 30,364, the ſines of incidence and refraction of ſuch light out of air into glaſs will conſequently be as 30,364 to 19, or what is equal to it, as 31,96 to 20 inſtead of 31 to 20, the ratio of the ſines of incidence and refraction, when the light enters the glaſs with its velocity undiminiſhed.

32. From hence a priſm, with a ſmall refracting angle, might perhaps be found to be no very inconvenient inſtrument for this purpoſe: for by ſuch a priſm, whoſe refracting angle was of one minute, for inſtance, the light with its velocity undiminiſhed would be turned out of its way 33″, and with the diminiſhed velocity 35″, 88 nearly, the difference between which being almoſt 2″. 53‴, would be the quantity by which the light, whoſe velocity was diminiſhed, would be turned out of its way more than that whoſe velocity was undiminiſhed.

33. Let us now be ſuppoſed to make uſe of ſuch a priſm to look at two ſtars, under the ſame circumſtances as the two ſtars in the example above-mentioned, the central one of which ſhould be large enough to diminiſh the velocity of its light one twentieth part, whilſt the velocity of the light of the other, which

which was ſuppoſed to revolve about it as a ſatellite, for want of ſufficient magnitude in the body from whence it was emitted, ſhould ſuffer no ſenſible diminution at all. Placing then the line, in which the two faces of the priſm would interſect each other, at right angles to a line joining the two ſtars; if the thinner part of the priſm lay towards the ſame point of the heavens with the central ſtar, whoſe light would be moſt turned out of its way, the apparent diſtance of the ſtars would be increaſed 2″. 53‴ and conſequently become 3″. 53‴ inſtead of 1″. only, the apparent diſtance ſuppoſed above in art. 21. On the contrary, if the priſm ſhould be turned half way round, and its thinner part lye towards the ſame point of the heavens with the revolving ſtar, their diſtance muſt be diminiſhed by a like quantity, and the central ſtar therefore would appear 1″. 53‴ diſtant from the other on the oppoſite ſide of it, having been removed from its place near three times the whole diſtance between them.

34. As a priſm might be made uſe of for this purpoſe, which ſhould have a much larger refracting angle than that we have propoſed, eſpecially if it was conſtructed in the achromatic way, according to Mr. DOLLOND's principles, not only ſuch a diminution, as one part in twenty, might be made ſtill more diſtinguiſhable; but we might probably be able to diſcover conſiderably leſs diminutions in the velocity of light, as perhaps a hundredth, a two-hundredth, a five-hundredth, or even a thouſandth part of the whole, which, according to what has been ſaid above, would be occaſioned by ſphæres, whoſe diameters ſhould be to that of the ſun, provided they were of the ſame denſity, in the ſeveral proportions nearly of 70, 50, 30, and 22 to 1 reſpectively.

35. If ſuch a diminution of the velocity of light, as that above ſuppoſed, ſhould be found really to take place, in conſe-

quence of its gravitation towards the bodies from whence it is emitted, and there ſhould be ſeveral of the fixed ſtars large enough to make it ſufficiently ſenſible, a ſet of obſervations upon this ſubject might probably give us ſome conſiderable information with regard to many circumſtances of that part of the univerſe, which is viſible to us. The quantity of matter contained in many of the fixed ſtars might from hence be judged of, with a great degree of probability, within ſome moderate limits; for though the exact quantity muſt ſtill depend upon their denſity, yet we muſt ſuppoſe the denſity moſt enormouſly different from that of the ſun, and more ſo, indeed, than one can eaſily conceive to take place in fact, to make the error of the ſuppoſed quantity of matter very wide of the truth, ſince the denſity, as has been ſhewn above in art. 11. and 12. which is neceſſary to produce the ſame diminution in the velocity of light, emitted from different bodies, is as the ſquare of the quantity of matter contained in thoſe bodies inverſely.

36. But though we might poſſibly from hence form ſome reaſonable gueſs at the quantity of matter contained in ſeveral of the fixed ſtars; yet, if they have no luminous ſatellites revolving about them, we ſhall ſtill be at a loſs to form any probable judgment of their diſtance, unleſs we had ſome analogy to go upon for their ſpecific brightneſs, or had ſome other means of diſcovering it; there is, however, a caſe that may poſſibly occur, which may tend to throw ſome light upon this matter.

37. I have ſhewn in my Enquiry into the probable Parallax, &c. of the Fixed Stars, publiſhed in the Philoſophical Tranſactions for the year 1767, the extremely great probability there is, that many of the fixed ſtars are collected together into groups; and that the Pleiades in particular conſtitute one of

theſe

theſe groups. Now of the ſtars which we there ſee collected together, it is highly probable, as I have obſerved in that paper, that there is not one in a hundred which does not belong to the group itſelf; and by far the greateſt part, therefore, according to the ſame idea, muſt lye within a ſphære, a great circle of which is of the ſame ſize with a circle, which appears to us to include the whole group. If we ſuppoſe, therefore, this circle to be about 2°. in diameter, and conſequently only about a thirtieth part of the diſtance at which it is ſeen, we may conclude, with the higheſt degree of probability, that by far the greateſt part of theſe ſtars do not differ in their diſtances from the ſun by more than about one part in thirty, and from thence deduce a ſort of ſcale of the proportion of the light which is produced by different ſtars of the ſame group or ſyſtem in the Pleiades at leaſt; and, by a ſomewhat probable analogy, we may do the ſame in other ſyſtems likewiſe. But having yet no means of knowing their real diſtance, or ſpecific brightneſs, when compared either with the ſun or with one another, we ſhall ſtill want ſomething more to form a farther judgment from.

38. If, however, it ſhould be found, that amongſt the Pleiades, or any other like ſyſtem, there are ſome ſtars that are double, triple, &c. of which one is a larger central body, with one or more ſatellites revolving about it, and the central body ſhould likewiſe be found to diminiſh the velocity of its light; and more eſpecially, if there ſhould be ſeveral ſuch inſtances met with in the ſame ſyſtem; we ſhould then begin to have a kind of meaſure both of the diſtance of ſuch a ſyſtem of ſtars from the earth, and of their mutual diſtances from each other. And if ſeveral inſtances of this kind ſhould occur in different groups or ſyſtems of ſtars, we might alſo, perhaps, begin to

form

form ſome probable conjectures concerning the ſpecific denſity and brightneſs of the ſtars themſelves, eſpecially if there ſhould be found any general analogy between the quantity of the diminution of the light and the diſtance of the ſyſtem deduced from it; as, for inſtance, if thoſe ſtars, which had the greateſt effect in diminiſhing the velocity of light ſhould in general give a greater diſtance to the ſyſtem, when ſuppoſed to be of the ſame denſity with the ſun, we might then naturally conclude from thence, that they are leſs in bulk, and of greater ſpecific denſity, than thoſe ſtars which diminiſh the velocity of light leſs, and *vice verſâ*. In like manner, if the larger ſtars were to give us in general a greater or leſs quantity of light in proportion to their bulk, this would give us a kind of analogy, from whence we might perhaps form ſome judgment of the ſpecific brightneſs of the ſtars in general; but, at all adventures, we ſhould have a pretty tolerable meaſure of the comparative brightneſs of the ſun and thoſe ſtars, upon which ſuch obſervations ſhould be made, if the reſult of them ſhould turn out agreeable to the ideas above explained.

39. Though it is not improbable, that a few years may inform us, that ſome of the great number of double, triple ſtars, &c. which have been obſerved by Mr. HERSCHEL, are ſyſtems of bodies revolving about each other, eſpecially if a few more obſervers, equally ingenious and induſtrious with himſelf could be found to ſecond his labours; yet the very great diſtance at which it is not unlikely many of the ſecondary ſtars may be placed from their principals, and the conſequently very long periods of their revolutions*, leave very little room to hope that

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* If the ſun, when removed to 10.000.000 times his preſent diſtance, would ſtill appear as bright as a ſtar of the ſixth magnitude, which I apprehend to be pretty

that any very great progreſs can be made in this ſubject for many years, or perhaps ſome ages to come; the above outlines, therefore, of the uſe that may be made of the obſervations upon the double ſtars, &c. provided the particles of light ſhould be ſubject to the ſame law of gravitation with other bodies, as in all probability they are, and provided alſo that ſome of the ſtars ſhould be large enough ſenſibly to diminiſh their velocity, will, I hope, be an inducement to thoſe, who may have it in their power, to make theſe obſervations for the benefit of future generations at leaſt, how little advantage ſoever we may expect from them ourſelves; and yet very poſſibly ſome obſervations of this ſort, and ſuch as may be made in a few years, may not only be ſufficient to do ſomething, even at preſent, but alſo to ſhew, that much more may be done hereafter, when theſe obſervations ſhall become more numerous, and have been continued for a longer period of years.

pretty near the truth, any ſatellite revolving round ſuch a ſtar, provided the ſtar was not either of leſs ſpecific brightneſs, or of greater denſity than the ſun, muſt, if it appeared at its greateſt elongation, at the diſtance of one ſecond only from its principal, be between three and four hundred years in performing one revolution; and the time of the revolution of the very ſmall ſtar near α Lyræ, if it is a ſatellite to this latter, and its principal is of the ſame ſpecific brightneſs and denſity with the ſun, could hardly be leſs than eight hundred years, though $37''$ the diſtance at which it is placed from it, according to Mr. Herschel's obſervations, ſhould happen to be its greateſt diſtance. Theſe periodical times, however, are computed from the above diſtances, upon the ſuppoſition of the ſtar, that revolves as a ſatellite, being very much ſmaller than the central one, ſo as not to diſturb its place ſenſibly; for if the two ſtars ſhould contain equal, or nearly equal, quantities of matter, the periodical times might be ſomewhat leſs, on account of their revolving about their common centre of gravity, in circles of little more than half as great a diameter as that in which the ſatellite muſt revolve upon the other ſuppoſition.